

Drag in a resonantly driven polariton fluid

A. C. Berceanu,^{*} E. Cancellieri[†], and F. M. Marchetti
*Departamento de Física Teórica de la Materia Condensada,
Universidad Autónoma de Madrid, Madrid 28049, Spain*
(Dated: March 4, 2013)

We study the linear response of a coherently driven polariton fluid in the pump-only configuration scattering against a point-like defect and evaluate analytically the drag force exerted by the fluid on the defect. When the system is excited near the bottom of the lower polariton dispersion, the sign of the interaction-renormalised pump detuning classifies the collective excitation spectra in three different categories [C. Ciuti and I. Carusotto, *physica status solidi (b)* **242**, 2224 (2005)]: linear for zero, diffusive-like for positive, and gapped for negative detuning. We show that both cases of zero and positive detuning share a qualitatively similar crossover of the drag force from the subsonic to the supersonic regime as a function of the fluid velocity, with a critical velocity given by the speed of sound found for the linear regime. In contrast, for gapped spectra, we find that the critical velocity exceeds the speed of sound. In all cases, the residual drag force in the subcritical regime depends on the polariton lifetime only. Also, well below the critical velocity, the drag force varies linearly with the polariton lifetime, in agreement with previous work [E. Cancellieri *et al.*, *Phys. Rev. B* **82**, 224512 (2010)], where the drag was determined numerically for a finite-size defect.

PACS numbers: 03.75.Kk, 71.36+c., 41.60.Bq

I. INTRODUCTION

Out of equilibrium quantum fluids such as polaritons in semiconductor microcavities are being the subject of an intensive study. Microcavity polaritons, the quasiparticles resulting from the strong coupling of cavity photons and quantum well excitons, have the prerogative of being easy to both manipulate, via an external laser, and detect, via the light escaping from the cavity [1]. In particular, resonant excitation allows the accurate tuning of the fluid properties, such as its density and current. However, the polariton lifetime being finite establishes the system as intrinsically out of equilibrium: An external pump is needed to continuously replenish the cavity of polaritons, that quickly, on a scale of tens of picoseconds, escape.

Recently, the superfluid properties of a resonantly pumped polariton quantum fluid in the pump-only configuration — i.e., where no other states aside the pump one are occupied by, e.g., parametric scattering — have been actively investigated both experimentally and theoretically [2–9]. This pumping scheme, differently from other cases, such as the resonant optical parametric oscillator regime and the non-resonant pumping scheme, creates a polariton fluid that, inside the pump spot, is not characterised by a free phase. On the contrary, the phase of the pump state is locked to the one of the external pumping laser. Nevertheless, it has been predicted [2, 3] and observed [4] that scattering can be suppressed below a critical velocity, where the system displays superfluid

behaviour, similarly to what has been predicted by the Landau criterion for equilibrium superfluid condensates. Further, a fixed phase clearly prevents the formation of phase dislocations, such as vortices and solitons. For this reason, it has been suggested [6] and experimentally realised [7] that the defect can be located just outside the pump spot, where the hydrodynamic nucleation of vortices, vortex-antivortex pairs, arrays of vortices, and solitons can be observed when the fluid collides with the extended defect. Similarly, nucleation of vortices in the wake of the obstacle has been observed in pulsed experiments [8, 9].

In a conservative quantum liquid flowing past a small defect, the Landau criterion for superfluidity links the onset of dissipation at a critical fluid velocity with the shape of the fluid collective excitation spectrum [10]. In particular, for weakly interacting Bose gases, the dispersion of the low-energy excitation modes being linear implies that the critical velocity for superflow coincides with the speed of sound c_s . Clearly, this is strictly correct only for vanishingly small perturbations [11], while for a defect with finite size and strength, the critical velocity can be smaller than c_s [12, 13].

However, even for perturbatively weak defects, in out-of-equilibrium systems, where the spectrum of excitations is complex, the validity of the Landau criterion has to be questioned [5, 14, 15]. In the particular case of coherently driven polaritons in the pump-only configuration, it has been predicted [2, 3], and later observed [4], that scattering is suppressed at either strong enough pump powers or small enough flow velocities. Yet, on a closer scrutiny, it has been shown that, despite the apparent validity of the Landau criterion, the system always experiences a residual drag force even in the limit of asymptotically large densities [5] or small velocities. This result has been proven by numerically solving the Gross-

[†]Present address: Laboratoire Kastler Brossel, Université Pierre et Marie Curie, École Normale Supérieure, CNRS, 4 place Jussieu, 75005 Paris, France

^{*}Corresponding author: andrei.berceanu@uam.es

Pitaevskii equation describing the resonantly-driven polariton system in presence of a non-perturbative extended defect. Here, the drag force exerted by the defect on the fluid has been shown to display a smooth crossover from the subsonic to the supersonic regime, similarly to what it has been found in the case of non-resonantly pumped polaritons [15]. In this work, we find an even richer phenomenology for the dependence of the drag force on the fluid velocity and two different kinds of crossovers from the sub- to the supercritical regime. Further, we show that the origin of the residual drag force, which, in agreement with Ref. [5], lies in the polariton lifetime only, can be demonstrated even within a linear response approximation.

More specifically, in this work, we apply the linear response theory to analytically evaluate the drag force exerted by the coherently driven polariton fluid in the pump-only configuration on a point-like defect. To simplify the formalism, we restrict our analysis to the case of resonant pumping close to the bottom of the lower polariton dispersion, where the dispersion is quadratic. Here, the properties of the collective excitation spectrum have been shown to be uniquely determined by three parameters only [3]: the fluid velocity v_p , the interaction-renormalised pump detuning Δ_p , and the polariton lifetime κ . In particular, the sign of the detuning Δ_p determines three qualitatively different types of spectra: linear for $\Delta_p = 0$, diffusive-like for $\Delta_p > 0$, and gapped for $\Delta_p < 0$.

For both cases of linear and diffusive spectra, we find a qualitatively similar behaviour of the drag force as a function of the fluid velocity v_p : In particular, the drag displays a crossover from a subsonic or superfluid regime — characterised by the absence of quasiparticle excitations — to a supersonic regime — where Cherenkov-like waves are generated by the defect and propagate into the fluid. The crossover becomes sharper for increasing polariton lifetimes $1/\kappa$ and displays the typical threshold behaviour for $\kappa \rightarrow 0$ with a critical velocity given by the speed of sound of linear regime, $v^c = c_s$, exactly as for weakly interacting equilibrium superfluids (in the case of perturbatively weak defects). This behaviour is similar to the one predicted for polariton superfluids excited non-resonantly [15], where the spectrum in that case is diffusive-like.

However, for gapped spectra at $\Delta_p < 0$, we find that the critical velocity governing the drag crossover exceeds the speed of sound, $v^c > c_s$, and we determine an analytical expression of v^c as a function of the detuning Δ_p . Further, for $\kappa \rightarrow 0$, the drag has a threshold-like behaviour qualitatively different from the one of weakly interacting equilibrium superfluids, with the drag jumping discontinuously from zero to a finite value at $v_p = v^c$.

We evaluate the drag as a function of the polariton lifetime κ and find for all three cases that: In the supercritical regime, $v_p > v^c$, the lifetime tends to suppress the propagation of the Cherenkov waves away from the defect and therefore to suppress the drag. Instead, well in

the subcritical regime, $v_p \ll v^c$, we find that the residual drag goes linearly to zero with the polariton lifetime κ , in agreement to what it was found in Ref. [5], by making use of a non-perturbative numerical analysis for a finite size defect. Similarly to Ref. [5], here, we do also find that the residual drag in the subcritical regime can be explained in terms of an asymmetric perturbation induced in the fluid by the defect in the direction of the fluid velocity.

This paper is structured as follows: In Sec. II we briefly introduce the linear response approximation. We classify the three types of collective excitation spectra in the simplified case of excitation close to the bottom of the lower polariton dispersion in Sec. II A. In Sec. III we derive the drag force and characterise the crossover from the subsonic to the supersonic regime in the three cases of zero, positive and negative detuning. In this section, we also evaluate the drag as a function of the polariton lifetime, interpreting therefore the results of Ref. [5]. Brief conclusions are drawn in Sec. IV.

II. LINEAR RESPONSE

The description of cavity polaritons resonantly excited by an external laser is usually formulated in terms of a classical non-linear Schrödinger equation (or Gross-Pitaevskii equation) [16] for the lower polariton (LP) field $\psi_{LP}(\mathbf{r}, t)$ ($\hbar = 1$):

$$i\partial_t\psi_{LP} = [\omega_{LP}(-i\nabla) - i\kappa + V(\mathbf{r}) + g|\psi_{LP}|^2]\psi_{LP} + \mathcal{F}(\mathbf{r}, t). \quad (1)$$

The LP dispersion is expressed in terms of the photon $\omega_C(\mathbf{k}) = \omega_C^0 + \frac{\mathbf{k}^2}{2m_C}$ and exciton ω_X^0 energies, the photon mass m_C , and the Rabi splitting Ω_R [1]:

$$\omega_{LP}(\mathbf{k}) = \frac{1}{2} [\omega_C(\mathbf{k}) + \omega_X^0] - \frac{1}{2} \sqrt{[\omega_C(\mathbf{k}) - \omega_X^0]^2 + \Omega_R^2}. \quad (2)$$

Because polaritons continuously decay at a rate κ , the cavity is replenished by a continuous wave resonant pump $F(\mathbf{r}, t)$ at a wavevector \mathbf{k}_p (we will later assume \mathbf{k}_p directed along the x -direction, $\mathbf{k}_p = (k_p, 0)$) and frequency ω_p :

$$\mathcal{F}(\mathbf{r}, t) = f_p e^{i(\mathbf{k}_p \cdot \mathbf{r} - \omega_p t)}. \quad (3)$$

Note that, as discussed in appendix A, Eq. (1) is a simplified description of the polariton system: This implies that the interaction non-linearities are small enough not to mix the lower and upper polariton branches. Moreover, starting from a formulation in terms of coupled exciton and photon fields, the polariton lifetime would be momentum dependent and, similarly, the polariton-polariton interaction strength g is not contact-like as instead assumed in Eq. (1). However, as shown in appendix A, these simplifications, do not affect our results

qualitatively, rather, allow to write them in terms of simpler expressions. Further, we have checked that, whenever the system is excited near the bottom of the lower polariton dispersion, the results for the drag force reported in Sec. III coincide with the ones obtained by using an exact photon-exciton coupled field description.

The potential $V(\mathbf{r})$ in Eq. (2) describes a defect, which can be either naturally present in the cavity mirror [4] or it can be created by an additional laser [17]. Later on, we will assume the defect to be point-like $V(\mathbf{r}) = g_V \delta(\mathbf{r})$ and weak, so that we can apply the linear response approximation [11]. In this treatment, one divides the response of the LP field in a mean-field component ψ_0 corresponding to the case when the perturbing potential is absent, and a fluctuation part $\delta\psi(\mathbf{r}, t)$ reflecting the linear response of the system to the perturbing potential:

$$\psi_{LP}(\mathbf{r}, t) = e^{-i\omega_p t} [e^{i\mathbf{k}_p \cdot \mathbf{r}} \psi_0 + \delta\psi(\mathbf{r}, t)] . \quad (4)$$

By substituting (4) into (1), we obtain a mean-field equation and, by retaining only the linear terms in the fluctuation field and the defect potential, the following first order equation in $\delta\psi(\mathbf{r}, t)$:

$$i\partial_t \begin{pmatrix} \delta\psi \\ \delta\psi^* \end{pmatrix} = \hat{\mathcal{L}} \begin{pmatrix} \delta\psi \\ \delta\psi^* \end{pmatrix} + V(\mathbf{r}) \begin{pmatrix} \psi_0 e^{i\mathbf{k}_p \cdot \mathbf{r}} \\ -\psi_0^* e^{-i\mathbf{k}_p \cdot \mathbf{r}} \end{pmatrix} , \quad (5)$$

where the operator $\hat{\mathcal{L}}$ is given by:

$$\hat{\mathcal{L}} = \begin{pmatrix} \widetilde{\omega_{LP}}(-i\nabla) - i\kappa & g\psi_0^2 e^{2i\mathbf{k}_p \cdot \mathbf{r}} \\ -g\psi_0^{*2} e^{-2i\mathbf{k}_p \cdot \mathbf{r}} & -\widetilde{\omega_{LP}}(-i\nabla) - i\kappa \end{pmatrix} , \quad (6)$$

with $\widetilde{\omega_{LP}} = \omega_{LP} - \omega_p + 2g|\psi_0|^2$. We are not interested here in solving the complex cubic mean-field equation for ψ_0 , as this has been already widely studied [1]. Rather, we want to study the response of the system to the presence of the defect and how different behaviours of the onset of dissipation can be described in terms of the different excitation spectra one can get for polaritons resonantly pumped close to the bottom of the LP dispersion.

A. Spectrum of collective excitations

The spectrum of the collective excitations can be obtained by diagonalising the operator $\hat{\mathcal{L}}$ in the momentum space representation:

$$\mathcal{L}_{\mathbf{k}, \mathbf{k}_p} = \begin{pmatrix} \widetilde{\omega_{LP}}(\delta\mathbf{k} + \mathbf{k}_p) - i\kappa & g\psi_0^2 \\ -g\psi_0^{*2} & -\widetilde{\omega_{LP}}(\delta\mathbf{k} - \mathbf{k}_p) - i\kappa \end{pmatrix} , \quad (7)$$

where, $\delta\mathbf{k} = \mathbf{k} - \mathbf{k}_p$. The description of the spectrum simplifies in the case when the pumping is close to the bottom of the LP dispersion, that can be approximated as parabolic

$$\omega_{LP}(\delta\mathbf{k} \pm \mathbf{k}_p) \simeq \omega_{LP}(0) + \frac{k_p^2}{2m} + \frac{\delta\mathbf{k}^2}{2m} \pm \delta\mathbf{k} \cdot \mathbf{v}_p , \quad (8)$$

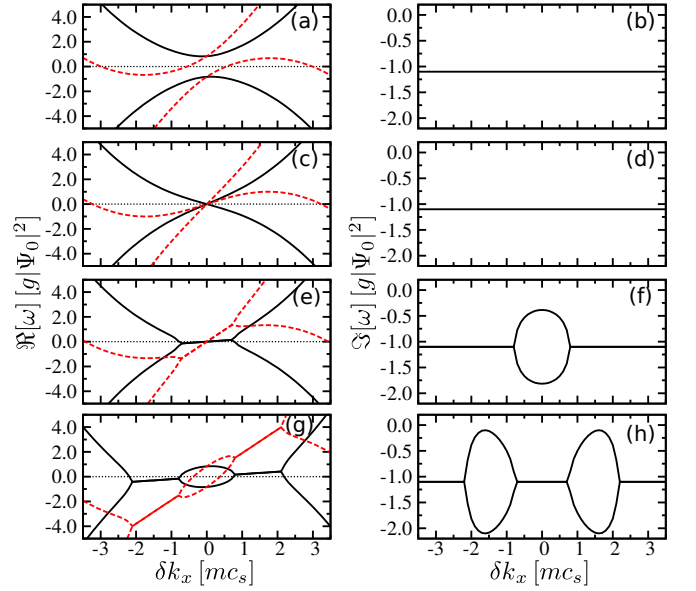


FIG. 1: (Color online) Collective excitation spectra for the subsonic (thick solid [black] line at $v_p = 0.2c_s$, with $c_s = \sqrt{g|\psi_0|^2/m}$) and supersonic (dashed [red] line at $v_p = 1.9c_s$) regimes and for an interaction-renormalised pump detuning $\Delta_p = -0.3g|\psi_0|^2$ (a, b), $\Delta_p = 0$ (c, d), $\Delta_p = 0.3g|\psi_0|^2$ (e, f) and $\Delta_p = 2.3g|\psi_0|^2$ (g, h). Real parts of the spectra are plotted in the left panels and the corresponding imaginary parts in the right panels for $\kappa = 1.1g|\psi_0|^2$ — note that in our description the spectrum imaginary parts do not depend on the fluid velocity v_p .

where $\mathbf{v}_p = \mathbf{k}_p/m$ is the fluid velocity, and m is the LP mass, $m = 2m_C[1 - (\omega_C^0 - \omega_X^0)/\sqrt{(\omega_C^0 - \omega_X^0)^2 + \Omega_R^2}]^{-1}$. This simplification allows one to describe the complex spectrum in terms of three parameters only, namely the fluid velocity \mathbf{v}_p , the interaction-renormalised pump detuning

$$\Delta_p = \omega_p - \left[\omega_{LP}(0) + \frac{k_p^2}{2m} + g|\psi_0|^2 \right] \quad (9)$$

and the LP lifetime κ :

$$\omega^\pm(\mathbf{k}) = \delta\mathbf{k} \cdot \mathbf{v}_p - i\kappa \pm \sqrt{\varepsilon(\delta\mathbf{k}) [\varepsilon(\delta\mathbf{k}) + 2g|\psi_0|^2]} , \quad (10)$$

where $\varepsilon(\mathbf{k}) = \frac{k^2}{2m} - \Delta_p$. If energies are measured in units of the mean-field energy blue-shift $g|\psi_0|^2$ (we will use the notation $\Delta'_p = \Delta_p/g|\psi_0|^2$ and $\kappa' = \kappa/g|\psi_0|^2$), then the fluid velocity v_p is measured in units of the speed of sound $c_s = \sqrt{g|\psi_0|^2/m}$. In order to make connection with the current experiments, note that, for blue-shifts in the range $g|\psi_0|^2 \simeq 0.1 - 1$ meV, typical values of the speed of sound c_s are $0.8 - 2.7 \times 10^6$ m/s. Similarly, for common values of the LP mass, the range in momenta in Fig. 1 comes of the order of $\delta k_x \simeq 0.2 - 0.8 \mu\text{m}^{-1}$.

The spectrum (10) can be classified according to the sign of the interaction-renormalised pump detuning

Δ_p [2, 3] — see Fig. 1. For $\Delta_p < 0$ [panels (a,b)], the real part of the spectrum is *gapped* while the imaginary part is determined by the polariton lifetime κ only. If one applies the Landau criterion making reference to the real part of the spectrum only, then one finds a critical velocity

$$\frac{v^c}{c_s} = \sqrt{1 + |\Delta'_p|} + \sqrt{|\Delta'_p|(|\Delta'_p| + 2)} > 1, \quad (11)$$

always larger than the speed of sound for $\Delta_p < 0$. If the fluid velocity is subcritical, $v_p < v^c$ (see [black] solid lines in Fig. 1(a)), then no quasiparticles can be excited and thus, for infinitely living polaritons $\kappa \rightarrow 0$, the fluid would experience no drag when scattering against the defect. For supercritical velocities instead, $v_p > v^c$ see [red] dashed lines in Fig. 1(a), one expects dissipation in the form of radiation of Cherenkov-like waves from the defect into the fluid. In the supercritical regime, the set of wavevectors \mathbf{k} for which $\Re[\omega^+(\mathbf{k})] = 0$ form a closed curve in the \mathbf{k} -space with no singularity of the derivative, i.e., in other words, the radiation can be emitted in all possible directions around the defect. This, as we will see in the next section, will imply that the drag force for $\kappa \rightarrow 0$ goes abruptly, rather than continuously, from zero at $v_p < v^c$ to a finite value at $v_p \geq v^c$.

The spectrum gap closes to zero in the resonant situation at $\Delta_p = 0$, when the two branches $\omega^\pm(\mathbf{k})$ touch at $\delta\mathbf{k} = 0$ [panels (c,d) of Fig. 1]: Here, the real part of the spectrum displays the standard *linear dispersion* at small wavevectors as for the weakly interacting bosonic gases, with the slope given by $c_s \pm v_p$. The imaginary part, as in the previous case, is constant and equal to $-\kappa$. It is clear therefore that in this case, when $\kappa \rightarrow 0$, one recovers the equilibrium results valid for weakly interacting gases [11, 18], where the critical velocity for superfluidity equals the speed of sound, $v^c = c_s$, and the drag displays a threshold like behaviour. Here, in the supersonic regime $v_p > v^c$, the close curve $\Re[\omega^+(\mathbf{k})] = 0$ has instead a singularity, resulting in the standard Mach cone of aperture θ , $\sin \theta = c_s/v_p$, inside which radiation from the defect cannot be emitted [18].

Finally, for $\Delta_p > 0$, the real parts of the particle $\omega^+(\mathbf{k})$ and hole $\omega^-(\mathbf{k})$ branches of the spectrum touch together in either one [$\Delta_p \leq 2$, see panels (e,f)] or two [$\Delta_p > 2$, see panels (g,h)] separate regions in momentum space. In the same regions, the corresponding imaginary parts instead split. With a somewhat abuse of language, we call these kinds of spectrum, *diffusive-like*. We note that, clearly, these spectra have no correspondence in equilibrium systems, because a finite polariton lifetime κ is needed in order for these modes to be stable, $\Im[\omega^\pm(\mathbf{k})] < 0$. We also note that for these spectra, even if considering only the real part of the collective excitation spectrum, as soon as the fluid is in motion $v_p > 0$, dissipation in the form of waves is possible. However, we will see that similarly to the case of polaritons non-resonantly pumped [15], when decreasing κ (and accordingly Δ_p in order to have stable solutions), this situation connects continuously to

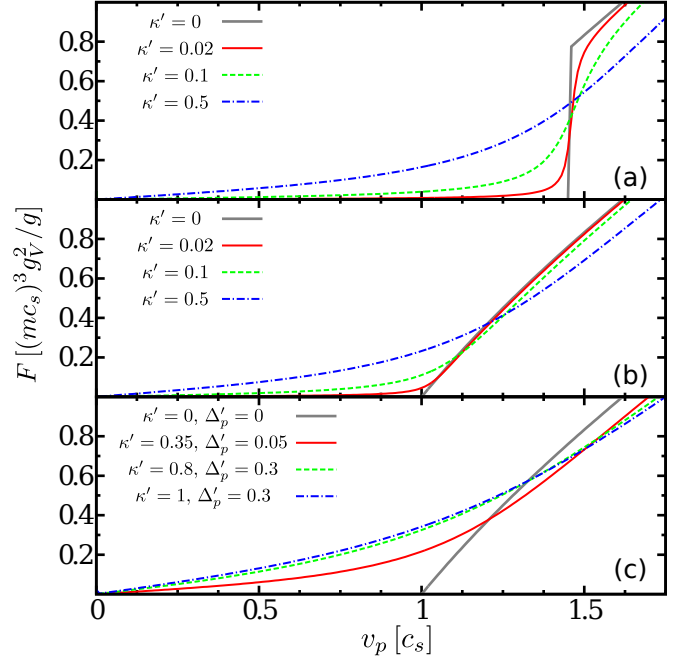


FIG. 2: (Color online) Drag force F as a function of the fluid velocity v_p for different values of the pump detuning Δ_p : $\Delta_p = -0.3g|\Psi_0|^2$ (a), $\Delta_p = 0$ (b), and $\Delta_p > 0$ (c), and for different values of the polariton lifetime — here, we use the notation $\kappa' = \kappa/g|\psi_0|^2$, $\Delta'_p = \Delta/g|\psi_0|^2$.

the previous case, where a threshold-like behaviour with $v^c = c_s$ was found.

We will see in the next section how these different spectra imply only two qualitatively different types of crossover of the drag force as a function of the fluid velocity, for either $\Delta_p < 0$ or $\Delta_p \geq 0$ pump detunings.

III. DRAG FORCE

The steady state response of the system to a static and weak defect can be evaluated starting from Eq. (5):

$$\begin{pmatrix} \delta\psi_s(\mathbf{r}) \\ \delta\psi_s^*(\mathbf{r}) \end{pmatrix} = \hat{\mathcal{L}}^{-1} \begin{pmatrix} V(\mathbf{r})e^{i\mathbf{k}_p \cdot \mathbf{r}}\psi_0 \\ -V(\mathbf{r})e^{-i\mathbf{k}_p \cdot \mathbf{r}}\psi_0^* \end{pmatrix}.$$

For a point-like defect, this can be written in momentum space as:

$$\delta\psi_s(\mathbf{k} + \mathbf{k}_p) = \frac{-g_V\psi_0(\varepsilon(\mathbf{k}) - \mathbf{k} \cdot \mathbf{v}_p + i\kappa)}{\varepsilon(\mathbf{k})[\varepsilon(\mathbf{k}) + 2g|\psi_0|^2] - (\mathbf{k} \cdot \mathbf{v}_p - i\kappa)^2},$$

while the other component $\delta\psi_s^*(\mathbf{k}_p - \mathbf{k})$ can be obtained by complex conjugation and by substituting $\mathbf{k} \mapsto -\mathbf{k}$. The drag force exerted by the defect on the fluid is given by [11]:

$$\mathbf{F} = - \int d\mathbf{r} |\psi_{LP}(\mathbf{r}, t)|^2 \nabla(V(\mathbf{r})), \quad (12)$$

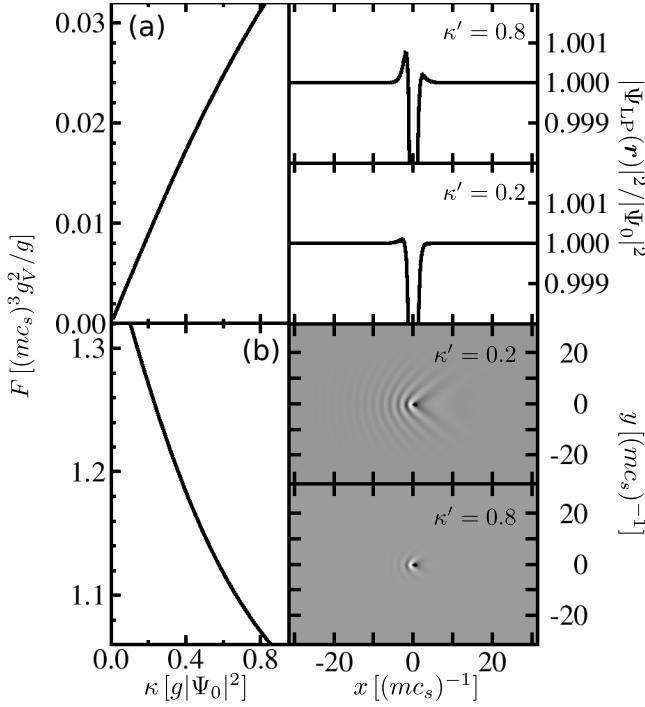


FIG. 3: (Color online) Drag force F as a function of the inverse polariton lifetime $\kappa' = \kappa/(g|\psi_0|^2)$ in the (a) subcritical regime ($v_p = 0.2c_s$) and (b) supercritical regime ($v_p = 1.9c_s$). In both cases we have fixed $\Delta_p = -0.3g|\psi_0|^2$ ($v^c \simeq 1.46c_s$) but these results are qualitatively similar for any other value of the pump detuning. We plot in the insets the normalised real-space wavefunction $|\psi_{LP}(\mathbf{r})|^2/|\psi_0|^2$ for two specific values of $\kappa' = 0.2$ and $\kappa' = 0.8$.

and, in the steady state linear response regime, we obtain:

$$\mathbf{F} = g_V \int \frac{d\mathbf{k}}{(2\pi)^2} i\mathbf{k} [\psi_0^* \delta\psi_s(\mathbf{k} + \mathbf{k}_p) + \psi_0 \delta\psi_s^*(\mathbf{k}_p - \mathbf{k})] \\ = 2g_V^2 |\psi_0|^2 \int \frac{d\mathbf{k}}{(2\pi)^2} \frac{i\mathbf{k}\varepsilon(\mathbf{k})}{\omega^+(\mathbf{k})\omega^-(\mathbf{k})}. \quad (13)$$

The drag is clearly oriented along the fluid velocity \mathbf{v}_p , i.e., $\mathbf{F} = F\hat{\mathbf{v}}_p$. If $\kappa \rightarrow 0$, then the integral in Eq. (13) is finite only if poles exist when $\Re[\omega^\pm(\mathbf{k})] = 0$, i.e., when quasiparticles can be excited, in agreement with the Landau criterion. For finite polariton lifetimes, however, it is clear that the integral will be always different from zero for $v_p > 0$. We now analyse the behaviour of the drag force as a function of the fluid velocity for the three ($\Delta_p = 0$, $\Delta_p > 0$, and $\Delta_p < 0$) different spectra illustrated in the previous section.

For the *linear* spectrum, at $\Delta_p = 0$, in the equilibrium limit, $\kappa \rightarrow 0$, we recover for the drag the known result of weakly interacting Bose gases in two dimensions [11]:

$$\frac{F}{(mc_s)^3 g_V^2/g} = \frac{(v_p/c_s)^2 - 1}{v_p/c_s} \Theta(v_p - c_s), \quad (14)$$

with a threshold-like behaviour at a critical fluid velocity equal to the speed of sound c_s . This limiting result is plotted as a bold gray line in the panels (b,c) of Fig. 2. For $\Delta_p = 0$ and finite lifetimes κ , we find a smooth crossover from the subsonic to the supersonic regime, with the drag being closer to the equilibrium threshold behaviour for decreasing κ (see Fig. 2(b)). A finite lifetime tends to increase the value of the drag in the subsonic region $v_p \ll v^c$, giving place to a residual drag force, similar to what was found in the numerical simulations of Ref. [5]. Instead, in the supersonic region $v_p \gg v^c$, the finite lifetime tends to decrease the value of the drag. In the case of *diffusive-like* spectra at $\Delta_p > 0$ the situation is qualitatively very similar to the resonant case (see Fig. 2(c)), with the difference that now, in order to have stable solutions, we can decrease the value of the lifetime only by decreasing accordingly also the value of the pump detuning Δ_p . The crossover for both $\Delta_p = 0$ and $\Delta_p > 0$ is also qualitatively very similar to the case of non-resonantly pumped polaritons [15], where the spectrum of excitation is in that case diffusive-like.

In the case of *gapped* spectra, the situation is however qualitatively different (see Fig. 2(a)). For infinitely living polaritons, $\kappa \rightarrow 0$, the drag force can also be evaluated analytically and its expression is similar to Eq. (14), but with a critical velocity larger than the speed of sound, which expression is given in Eq. (11):

$$\frac{F}{(mc_s)^3 g_V^2/g} = \frac{(v_p/c_s)^2 - 1}{v_p/c_s} \Theta(v_p - v^c). \quad (15)$$

Therefore now the drag experiences a jump for $v_p = v^c$, rather than a continuous threshold as for the resonant case $\Delta_p = 0$. As already mentioned in the previous section, this discontinuous behaviour of the drag for the gapped spectra is connected to the fact that, as soon as quasiparticles can be excited by the defect at $v_p \geq v^c$, Cherenkov-like waves can be immediately emitted in all directions, rather than being restricted in a region outside the Mach cone like before. For $\Delta_p = 0$, the cone was gradually closing with increasing the fluid velocity.

Both the increase of the value of the drag in the subcritical region as a function of the polariton lifetime and the decrease in the supercritical region, are behaviours common to all the types of spectra. We plot the drag force as a function of κ in Fig. 3, for two values of the fluid velocity v_p and a specific value of the pump detuning Δ_p , though we have checked that the following results are generic. For $v_p < v^c$, we find that the residual drag is a finite-lifetime effect only, and, in agreement with the results of Ref. [5], we find that, well below the critical velocity, the drag force goes linearly to zero for $\kappa \rightarrow 0$. In the resonant case $\Delta_p = 0$, the slope of the drag for $v_p \ll c_s$ can be evaluated analytically starting from the expression (13):

$$\frac{F}{(mc_s)^3 g_V^2/g} \underset{\kappa \rightarrow 0}{\simeq} \frac{2c_s}{\pi v_p} \left(\frac{1}{\sqrt{1 - (v_p/c_s)^2}} - 1 \right) \frac{\kappa}{g|\psi_0|^2}.$$

The residual drag in the subsonic regime is an effect of the broadening of the quasi-particles energies: Even when the spectrum real part does not allow any scattering against the defect (e.g., for $\Delta_p \leq 0$), the broadening produces some scattering close to the defect. This results in a perturbation of the fluid around the defect, asymmetric in the direction of the fluid velocity (see panel (a) of Fig. 3), similarly to what it was obtained in Ref. [5]. Instead, in the supersonic regime, the drag force is weaker in the non-equilibrium case respect to the equilibrium one. This is caused by the finite lifetime tending to suppress the propagation of the Cherenkov waves away from the defect, as shown in panel (b) of Fig. 3.

IV. CONCLUSIONS AND DISCUSSION

To conclude, we have analysed the linear response to a weak defect of resonantly pumped polaritons in the pump-only state and we have been able to determine two different kinds of threshold like behaviours for the drag force as a function of the fluid velocity. In the case of either zero or positive pump detuning, one can continuously connect to the case of equilibrium weakly interacting gases, where the drag displays a continuous threshold with a critical velocity equal to the speed of sound. However, for negative pump detuning, where the spectrum of excitations is gapped, the drag shows a discontinuity with a critical velocity larger than the speed of sound. In this sense, the case of coherently driven micro-cavity polaritons in the pump-only configuration displays a richer phenomenology than the case of polariton superfluids non resonantly pumped. It would be interesting to perform a similar analysis in the case of polaritons in the optical parametric oscillator regime, where polaritons are parametrically scattered from the pump state to the signal and idler states. Here, the spectrum of excitations has been already determined in Ref. [19], however it is far from clear what are the conditions for subcritical, superfluid, behaviour in a fluid characterised by three distinct currents, and how the link between signal and idler imposed by the parametric scattering influences the scattering of both fluids against a defect.

Acknowledgments

We are grateful to C. Tejedor and M. Szyman-ska for useful discussions. The authors acknowledge the financial support from the Spanish MINECO (MAT2011-22997), CAM (S-2009/ESP-1503), FP7 ITN "Clermont4" (A.B.), and from the program Ramón y Cajal (F.M.M.).

Appendix A: Gross-Pitaevskii equation for the lower polariton field

If one starts from a descriptions of polaritons in terms of separate exciton and cavity photon fields, a rotation into the lower and upper polariton basis, followed by neglecting the occupancy of the upper polariton branch, results in the following Gross-Pitaevskii equation for the lower polariton (LP) field in momentum space $\psi_{LP}(\mathbf{r}, t) = \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \psi_{LP, \mathbf{k}}(t)$ [16]:

$$i\partial_t \psi_{LP, \mathbf{k}} = f_p e^{-i\omega_p t} \delta_{\mathbf{k}, \mathbf{k}_p} + [\omega_{LP}(k) - i\kappa(k)] \psi_{LP, \mathbf{k}} + \sum_{\mathbf{k}_1, \mathbf{k}_2} g_{\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2} \psi_{LP, \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}}^* \psi_{LP, \mathbf{k}_1} \psi_{LP, \mathbf{k}_2} + s_k \sum_{\mathbf{k}_1} V_{\mathbf{k} - \mathbf{k}_1} \psi_{LP, \mathbf{k}_1} s_{k_1}, \quad (\text{A1})$$

where $\kappa(k) = \kappa_X c_k^2 + \kappa_C s_k^2$ is the effective LP decay rate, $g_{\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2} = g_X c_k c_{|\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}|} c_{k_1} c_{k_2}$ is the interaction strength, and where $V(\mathbf{r}) = \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} V_{\mathbf{k}}$. In these expressions, the coefficients

$$c_k^2, s_k^2 = \frac{1}{2} \left(1 \pm \frac{\omega_C(k) - \omega_X^0}{\sqrt{(\omega_C(k) - \omega_X^0)^2 + \Omega_R^2}} \right) \quad (\text{A2})$$

are the Hopfield coefficients used to diagonalise the free polariton Hamiltonian. We want here to justify the simplified description done in Eq. (1). If we follow the linear response expansion as in (4), the operator $\hat{\mathcal{L}}$ in momentum space analogous to (7) reads as:

$$\mathcal{L}_{\mathbf{k}, \mathbf{k}_p} = \begin{pmatrix} \widetilde{\omega_{LP}}(\delta\mathbf{k} + \mathbf{k}_p) - i\kappa(\delta\mathbf{k} + \mathbf{k}_p) & g_X c_{k_p}^2 c_{\delta\mathbf{k} + \mathbf{k}_p} c_{\delta\mathbf{k} - \mathbf{k}_p} \psi_0^2 \\ -g_X c_{k_p}^2 c_{\delta\mathbf{k} + \mathbf{k}_p} c_{\delta\mathbf{k} - \mathbf{k}_p} \psi_0^{*2} & -\widetilde{\omega_{LP}}(\delta\mathbf{k} - \mathbf{k}_p) - i\kappa(\delta\mathbf{k} - \mathbf{k}_p) \end{pmatrix}, \quad (\text{A3})$$

where now $\widetilde{\omega_{LP}}(\delta\mathbf{k} \pm \mathbf{k}_p) = \omega_{LP}(\delta\mathbf{k} \pm \mathbf{k}_p) - \omega_p + 2g_X c_{k_p}^2 c_{\delta\mathbf{k} \pm \mathbf{k}_p}^2 |\psi_0|^2$. It is easy to show that the eigenvalues of this operator coincide with our approximated expressions (10) in the limit of $\delta k \ll k_p$, when $c_{\delta\mathbf{k} \pm \mathbf{k}_p}^2 \simeq c_{k_p}^2$, $s_{\delta\mathbf{k} \pm \mathbf{k}_p}^2 \simeq s_{k_p}^2$ and when we can simply rename $g = g_X c_{k_p}^4$ and $\kappa = \kappa(k_p)$. It is interesting to note that, even if we

would retain the linear terms in $\mathbf{k}_p \cdot \delta\mathbf{k}$ in the expansion of $c_{\delta\mathbf{k} \pm \mathbf{k}_p}^2$, this would result in a renormalisation of the fluid velocity \mathbf{v}_p in the expression (10) which takes into account the blue-shift of the lower polariton dispersion due to the interaction.

-
- [1] A. V. Kavokin, J. J. Baumberg, G. Malpuech, and F. P. Laussy, *Microcavities* (Oxford University Press, Oxford, 2007).
 - [2] I. Carusotto and C. Ciuti, Phys. Rev. Lett. **93**, 166401 (2004).
 - [3] C. Ciuti and I. Carusotto, physica status solidi (b) **242**, 2224 (2005).
 - [4] A. Amo, J. Lefrère, S. Pigeon, C. Adrados, C. Ciuti, I. Carusotto, R. Houdré, E. Giacobino, and A. Bramati, Nat. Phys. **5**, 805 (2009).
 - [5] E. Cancellieri, F. M. Marchetti, M. H. Szymańska, and C. Tejedor, Phys. Rev. B **82**, 224512 (2010).
 - [6] S. Pigeon, I. Carusotto, and C. Ciuti, Phys. Rev. B **83**, 144513 (2011).
 - [7] A. Amo, S. Pigeon, D. Sanvitto, V. G. Sala, R. Hivet, I. Carusotto, F. Pisanello, G. Leménager, R. Houdré, E. Giacobino, et al., Science **332**, 1167 (2011).
 - [8] G. Nardin, G. Grosso, Y. Leger, B. Pietka, F. Morier-Genoud, and B. Deveaud-Plédran, Nature Physics **7**, 635 (2011).
 - [9] D. Sanvitto, S. Pigeon, A. Amo, D. Ballarini, M. D. Giorgi, I. Carusotto, R. Hivet, F. Pisanello, V. G. Sala, P. S. Soares-Guimaraes, et al., Nature Physics **6**, 527 (2011).
 - [10] L. P. Pitaevskii and S. Stringari, *Bose-Einstein Condensation* (Clarendon Press, Oxford, 2003).
 - [11] G. E. Astrakharchik and L. P. Pitaevskii, Phys. Rev. A **70**, 013608 (2004).
 - [12] R. Onofrio, C. Raman, J. M. Vogels, J. R. Abo-Shaeer, A. P. Chikkatur, and W. Ketterle, Phys. Rev. Lett. **85**, 2228 (2000).
 - [13] S. Iasenelli, C. Menotti, and A. Smerzi, J. Phys. B: At. Mol. Opt. Phys. **39**, S135 (2006).
 - [14] M. H. Szymańska, J. Keeling, and P. B. Littlewood, Phys. Rev. Lett. **96**, 230602 (2006).
 - [15] M. Wouters and I. Carusotto, Phys. Rev. Lett. **105**, 020602 (2010).
 - [16] C. Ciuti, P. Schwendimann, and A. Quattropani, Semicond. Sci. Technol. **18**, S279 (2003).
 - [17] A. Amo, S. Pigeon, C. Adrados, R. Houdré, E. Giacobino, C. Ciuti, and A. Bramati, Phys. Rev. B **82**, 081301 (2010).
 - [18] I. Carusotto, S. X. Hu, L. A. Collins, and A. Smerzi, Phys. Rev. Lett. **97**, 260403 (2006).
 - [19] M. Wouters and I. Carusotto, Phys. Rev. A **76**, 043807 (2007).